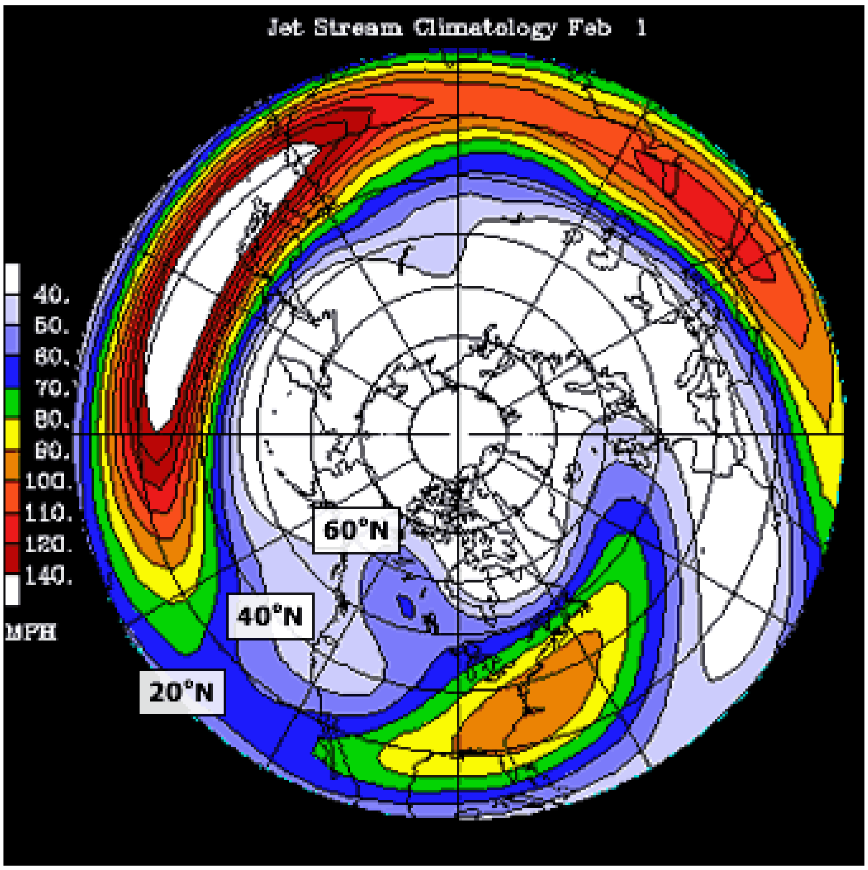
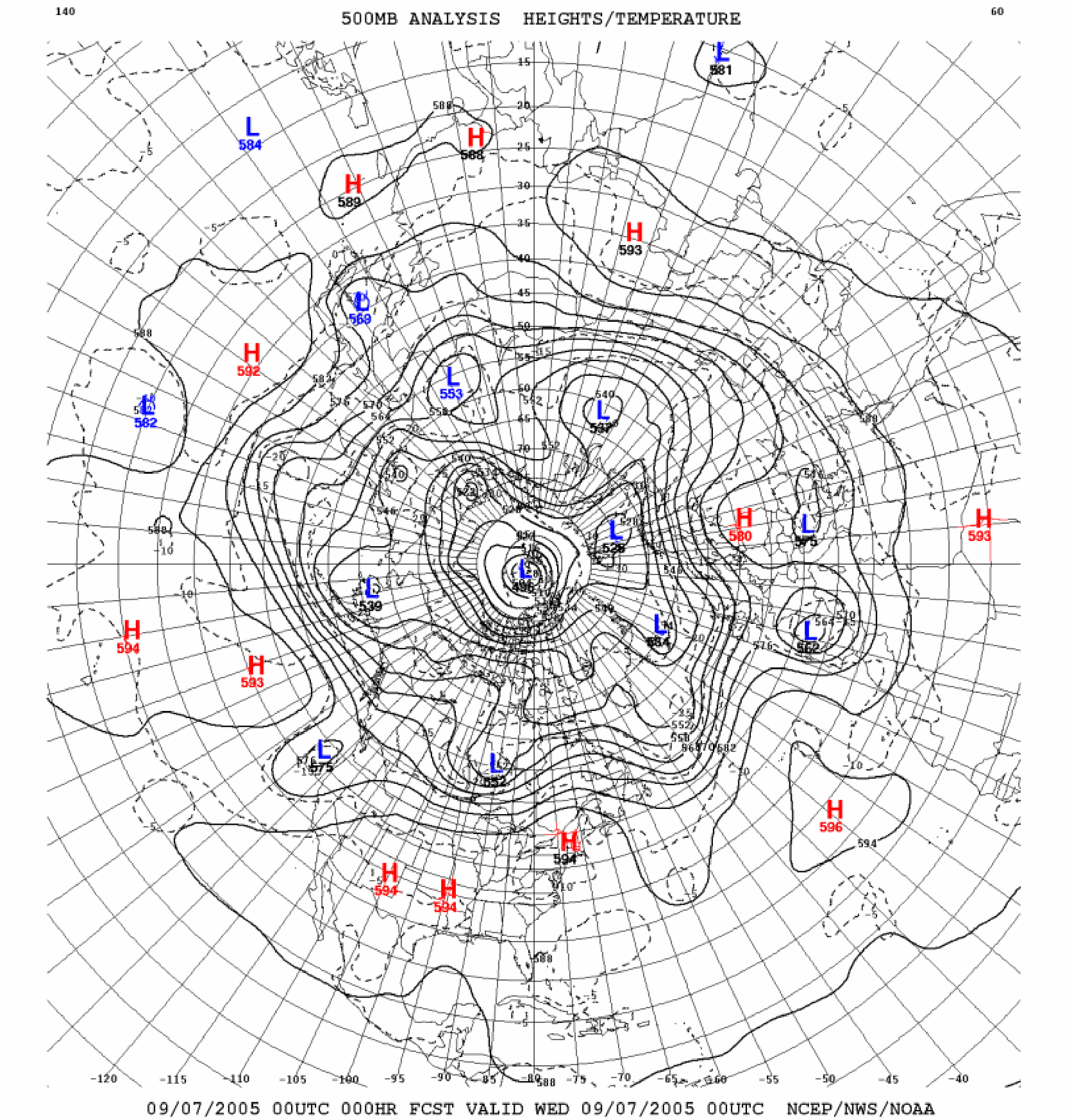
2. Baroclinic Instability and Midlatitude Dynamics



Midlatitude Jet Stream Climatology (Atlantic and Pacific)

Eddies

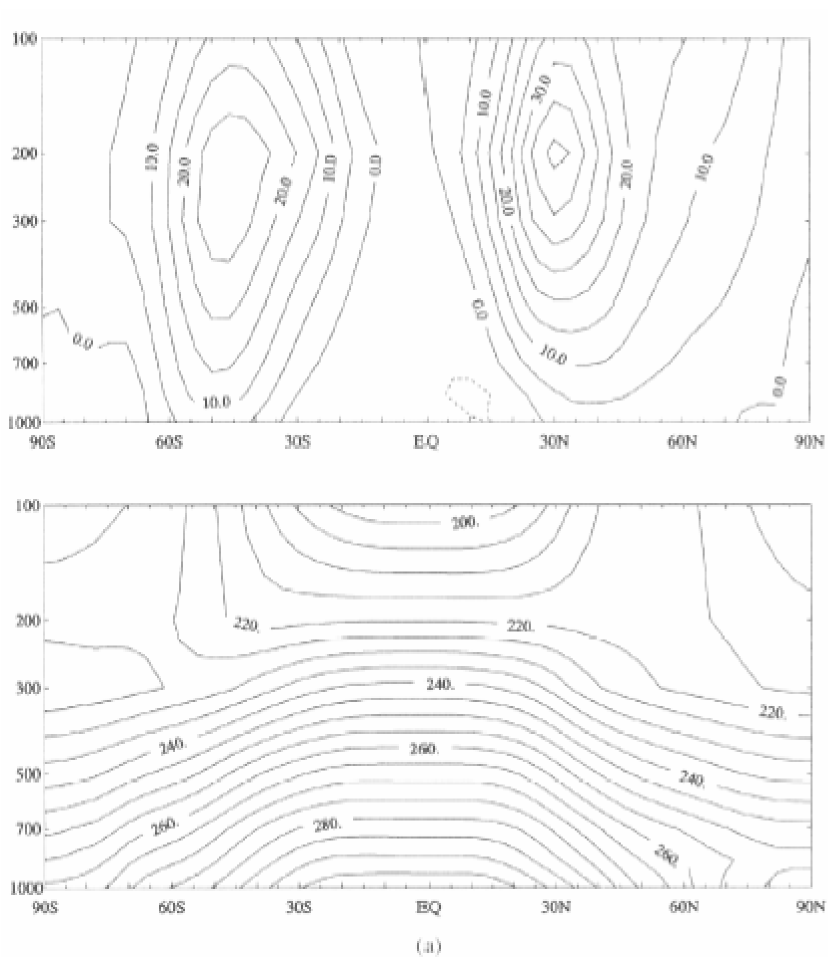


What sets the scale of eddies? What is the mechanism for their development?

Instabilities

Rotating fluid adjusts to **geostrophic equilibrium**: has potential energy available for conversion to other forms. Examine small disturbances: do dynamical constraints allow the disturbances to draw on APE?

Basic midlaitude flow in lower atmosphere is jetstream with both horizontal and vertical mean-flow shears.



wind

temperature

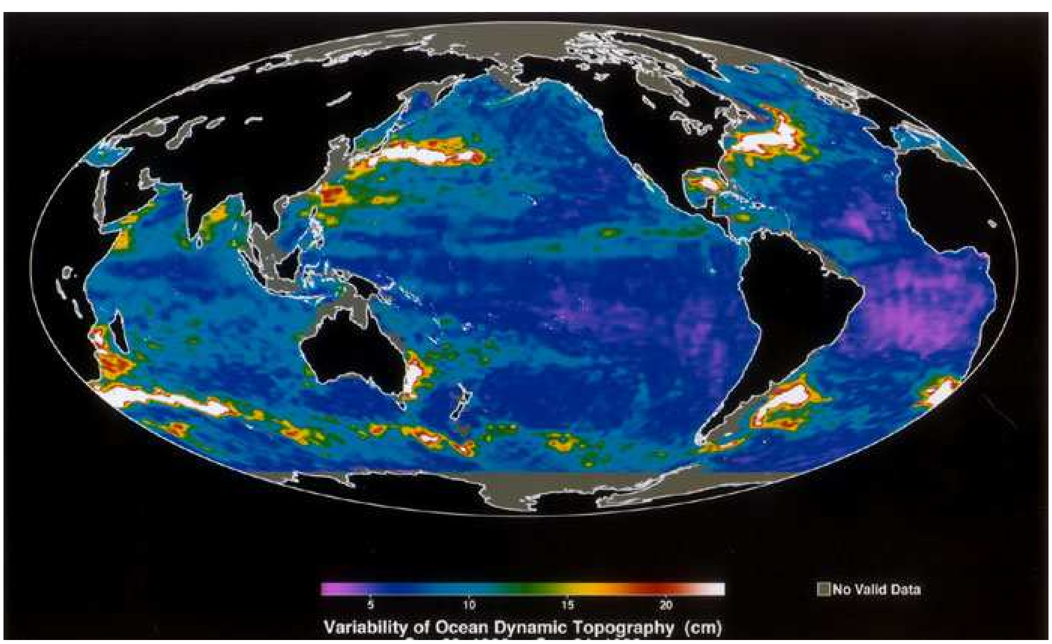
Fig. 6.1 Meridional cross sections of longitudinally averaged zonal wind (top panels, m s^{-1}) and temperature (bottom panels, contours, K) for (a) DJF and (b) JJA averaged for years 1980–1987. (Adapted from Schubert *et al.*, 1990.) (*Figure continues.*)

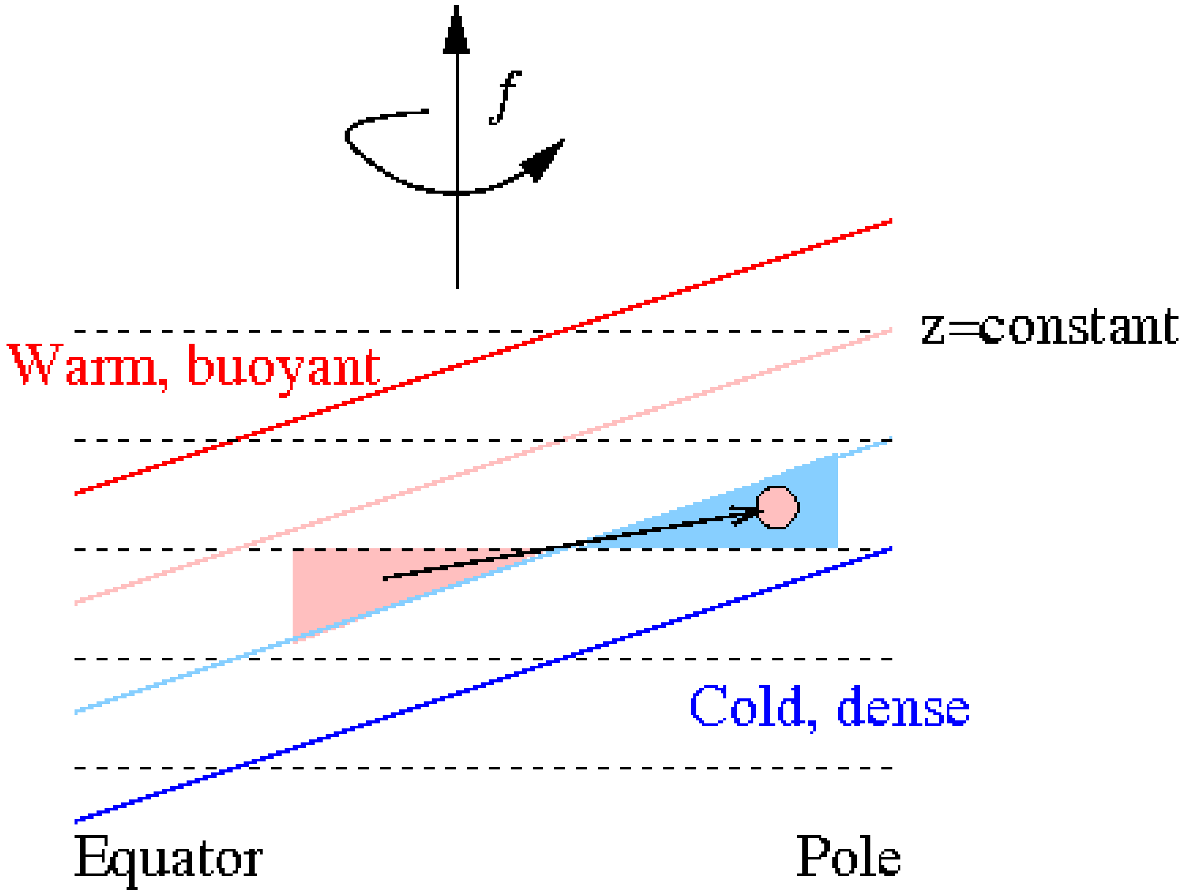
Barotropic instability: wave instability associated with horizontal shear. Extracts KE from mean flow.

Baroclinic instability: wave instability associated with vertical shear. Converts PE associated with mean horizontal temp gradient that must exist to provide thermal wind balance for vertical shear.

Wave instabilities important for synoptic-scale meteorology are zonally asymmetric perturbations (**eddies**) to zonally symmetric flow field.

Baroclinic instabilities also observed in ocean - often more complex since the flow is rarely purely zonal.





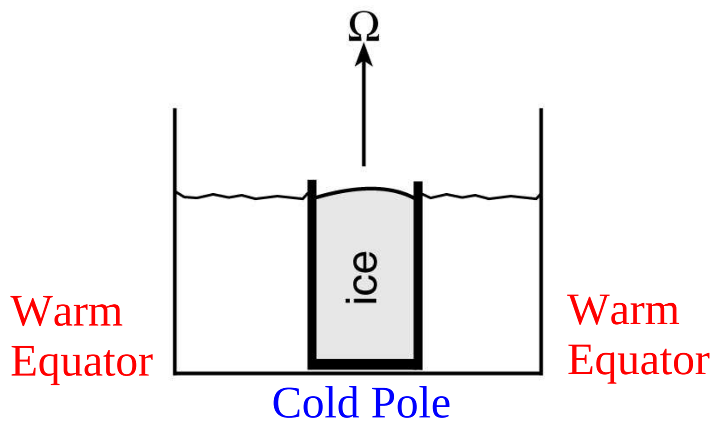
But, having PE available in flow is not sufficient for instability - rotation tends to inhibit release of PE.

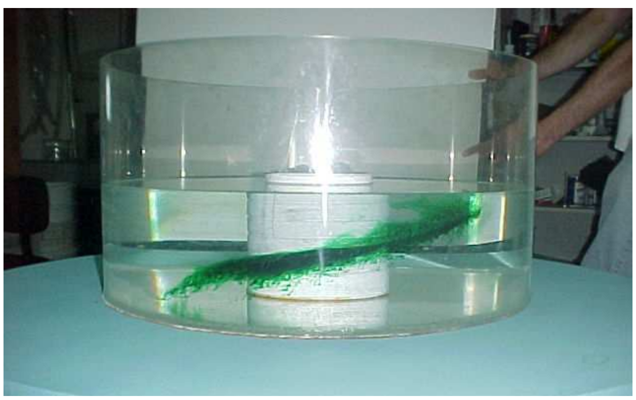
Analogy with shear instability of 2-D vortex dynamics – always KE available, but flow isn't always unstable.

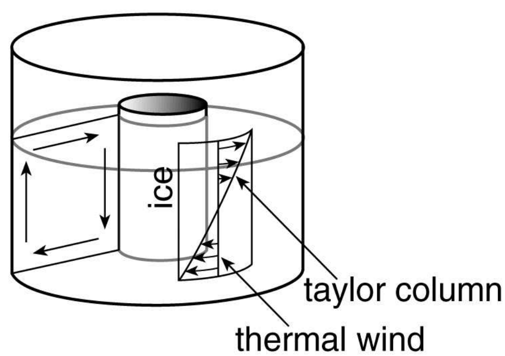
How can the energy be released?

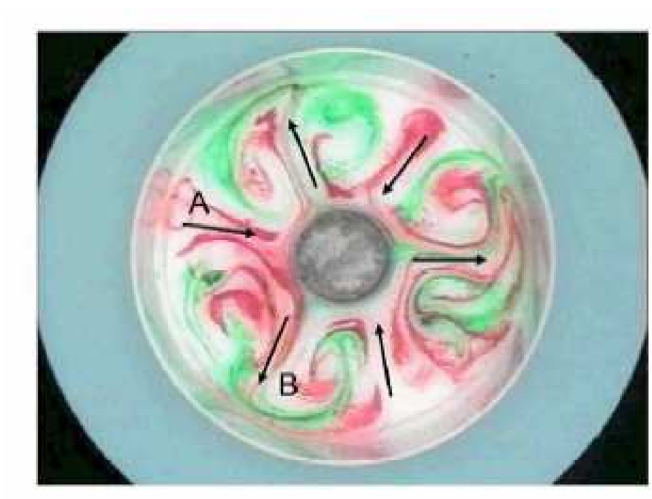
Consider simple model that contains the important feature – vertical shear (and hence horizontal density contrasts).

Laboratory model









Formation of eddies

The Eady problem

Eady problem: disturbances to westerly sheared jet, meridional temperature gradient. Stability properties? A simple paradigm for baroclinic instability.

Analyse structure (**eigenfunctions**) and growth rates (**eigenvalues**) for unstable modes in simplest possible model that satisfies necessary conditions for instability.

The model (Eady, 1949; see also Charney 1947):

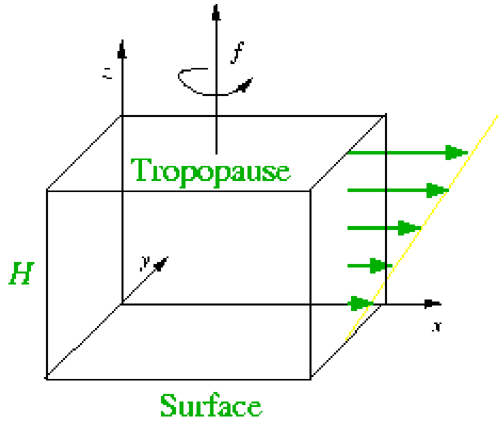
Boussinesq approximation (basic state density constant)

f-plane approximation, $\beta = 0$

$\partial \bar{u} / \partial z = \Lambda = \text{constant}$

rigid lid at $z = 0, H$

N^2 taken to be constant



Constant rotation $f \equiv f_0$.

Simplifications not realistic for atmosphere, or (except constant density) for ocean. But model just rich enough to expose fundamental nature of the instability.

[Infinite region of large static stability N^2 in place of upper lid gives only qualitative changes in results.]

Use quasi-geostrophic potential vorticity (QGPV) equation: $\frac{D_g}{Dt} q = 0$ where $q = \nabla^2 \psi + \frac{f}{N^2} \frac{\partial^2 \psi}{\partial z^2}$, and the thermodynamic equation: $\frac{D_g}{Dt} \frac{\partial \psi}{\partial z} + \frac{N^2}{f} w = 0$.

For basic state with $\overline{\psi} = -\Lambda z y$, potential vorticity is zero $\overline{q} = 0$.

Linearised QGPV equation is: $\left(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{\partial x}\right)q'=0,$

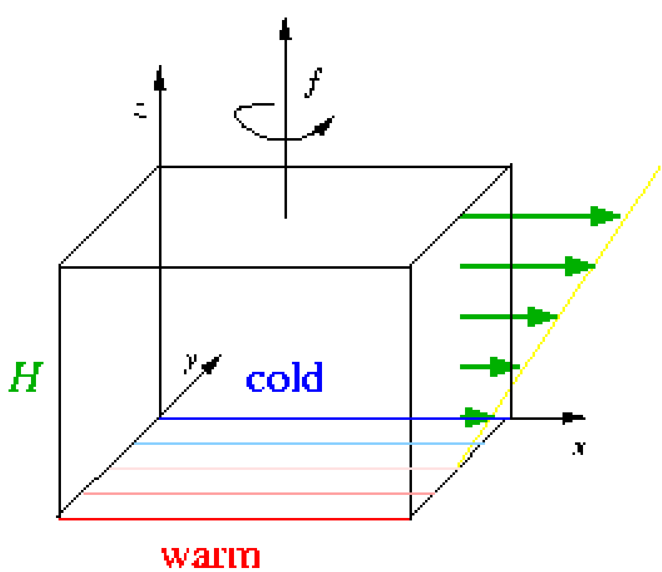
implies QGPV of disturbance q' is zero in interior.

Thus dynamics set by boundary conditions. Without upper boundary flow would be stable.

Linearised thermodynamic equation at a boundary where $w'=0$ is: $\left(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{\partial x}\right)\left(\frac{f_0}{N^2}\frac{\partial\psi'}{\partial z}\right)-\frac{f_0}{N^2}\frac{\partial\psi'}{\partial x}\frac{\partial\bar{u}}{\partial z}=0.$

Vertical shear at upper & lower boundaries implies temperature gradient (in cross-flow direction).

Advection of temperature field at upper & lower boundaries must therefore be taken into account.



In Eady problem evolution of flow controlled by temperature at upper & lower boundaries

Eady edge waves

Consider lower boundary only. Equations become:

$$\psi'_{xx} + \psi'_{yy} + \frac{f_0}{N^2}\psi'_{zz} = 0 \text{ and } \frac{\partial}{\partial t}\left(\frac{f_0}{N^2}\psi'_z\right) - \frac{\Lambda f_0}{N^2}\psi'_x = 0.$$

Look for solutions with $\psi' = \mathrm{Re}[\varphi(z)e^{ikx+ily-i\omega t}]$.

Dispersion reln: $\omega = \dfrac{k\Lambda f_0}{N(k^2+l^2)^{1/2}}$

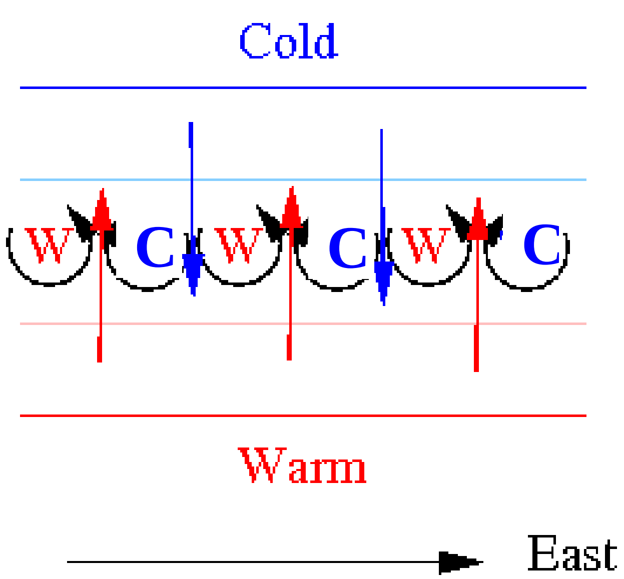
eastward phase speed

Consider upper boundary only.

Dispersion reln: $\omega = -\dfrac{k\Lambda f_0}{N(k^2+l^2)^{1/2}} + k\Lambda H$

westward phase speed relative to flow

Warm anomalies on lower (*upper*) level: (*anti*)cyclonic flow, propagate to east (*west relative to mean flow*).



But no heat is transported poleward and no energy is released.

Lower boundary

Both boundaries included: $\psi' = \mathrm{Re}[\varphi(z)e^{ikx+ily-i\omega t}]$

$\varphi(z) = A\cosh\kappa z + B\sinh\kappa z$ where $\kappa = (k^2+l^2)^{1/2}\dfrac{N}{f_0}$

Boundary conditions $\left.\begin{array}{l}\psi'_{zt} - \Lambda\psi'_x = 0 \\ \psi'_{zt} + \Lambda H\psi'_x = 0\end{array}\right\}$ give A, B.

Dispersion relation: $\omega = k\Lambda H\left(\dfrac{1}{2} \pm \sqrt{\alpha}\right)$ where

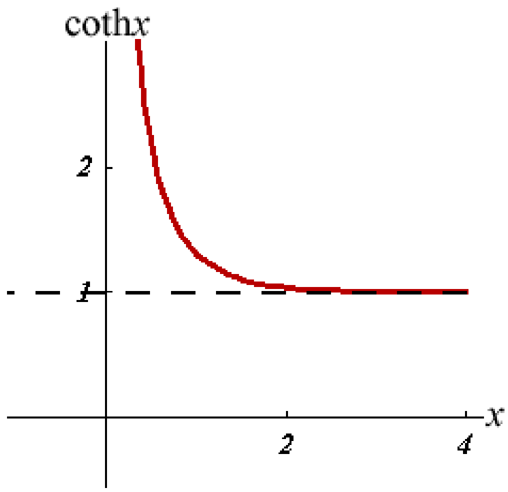
$$\alpha = \left[\frac{1}{4} - \frac{\coth\kappa H}{\kappa H} + \frac{1}{\kappa^2 H^2}\right]$$

ω real or complex depending on sign of α.

When $\alpha < 0$ flow is baroclinically unstable. When $\alpha = \alpha_c = 0$ flow is neutrally stable.

Hence instability requires $\alpha < \alpha_c$ or $\left(k^2 + l^2\right) < \alpha_c^2 f_0^2 / N^2 \approx 5.76 / L_R^2$ where $L_R = NH / f_0 \approx 1000\mathrm{km}$ is Rossby radius of deformation.

1. Deep domain/ short waves ($\kappa H >> 1$)



$$\omega \approx k\Lambda H\left(\frac{1}{2} \pm \frac{1}{2}\left[1+\frac{4}{\kappa H}\right]^{1/2}\right) \approx k\Lambda H\left(\frac{1}{2} \pm \left[\frac{1}{2} - \frac{1}{\kappa H}\right]\right)$$

$$\omega_- \approx \frac{k\Lambda}{\kappa}$$ ← eastward wave (lower boundary)

$$\omega_+ \approx \kappa\Lambda H - \frac{k\Lambda}{\kappa}$$ ← westward wave relative to flow (upper boundary)

2. Shallow domain /long waves ($\kappa H << 1$)

$\alpha < 0$, ω is complex instability. Maximum growth rate for smallest κ, i.e. when $l = 0$.

Insights from the Eady problem

1. Wavenumber k for no growth $2.40\frac{f_0}{NH} = 2.40 / L_R$

2. In channel with rigid walls, instability occurs if channel width $> 1.31\frac{NH}{f_0} = 1.31 L_R$. Current must be at least as broad as deformation radius for instability.

3. Maximum growth rate occurs at $l = 0$ (no cross-flow structure). Max growth rate $0.31\frac{f_0 \Lambda}{N}$.

4. Wavenumber k for max growth $1.61\frac{f_0}{NH} = 1.61 / L_R$.

5. Length scale (quarter wavelength) of fastest growing mode just under L_R. (Wavelength λ is $\lambda = 2\pi / k$).

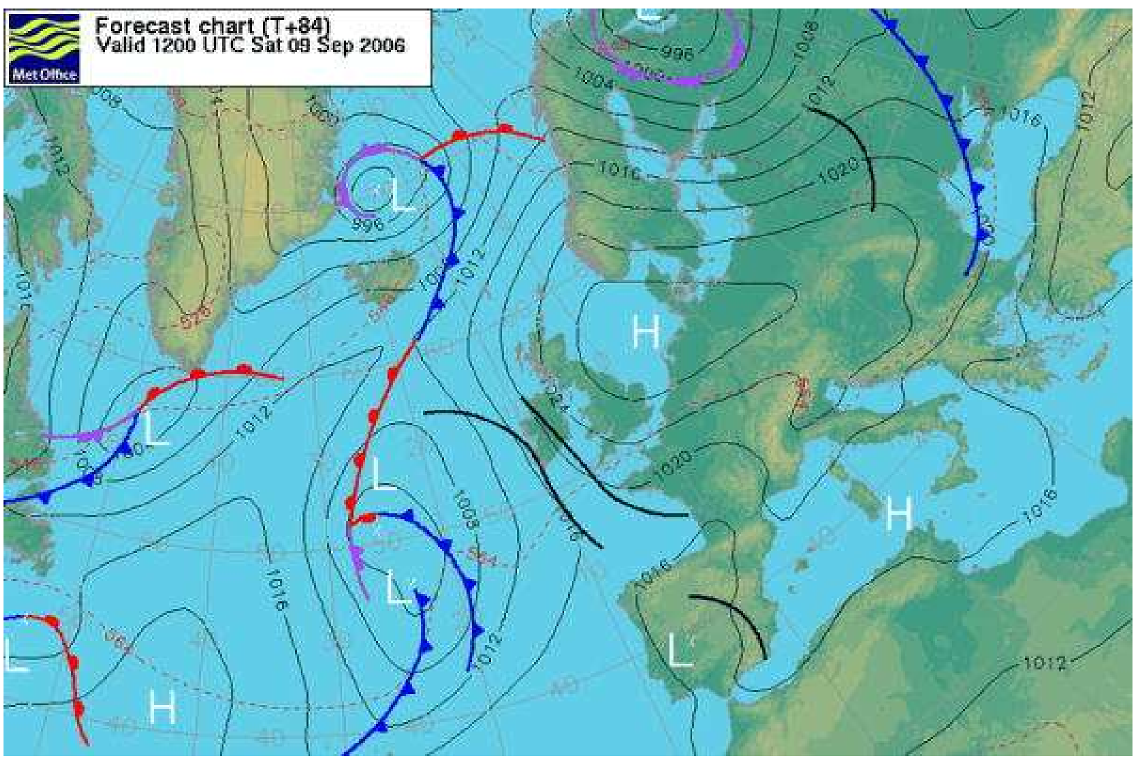
Thus expect to see **unstable disturbances with scales of order the deformation radius**.

This is fundamental reason why both atmosphere and ocean are filled with perturbations with horizontal spatial scales of order the deformation radius.

Atmosphere: $\frac{NH}{f_0} = L_R \sim 1000\text{km}$ & $\frac{f_0 \Lambda}{N} \sim 5\times10^{-5}\text{s}^{-1}$

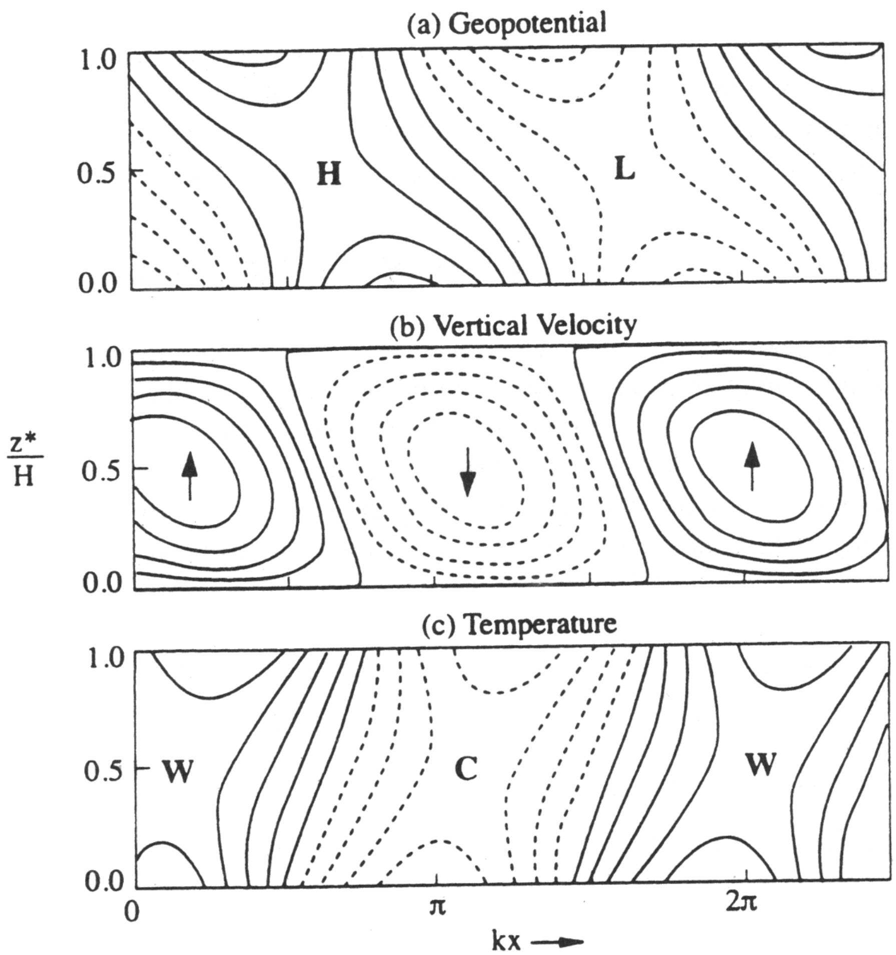
e-folding time for fastest growing waves: 18 hours

Wavelength of fastest growing waves: 4000 km



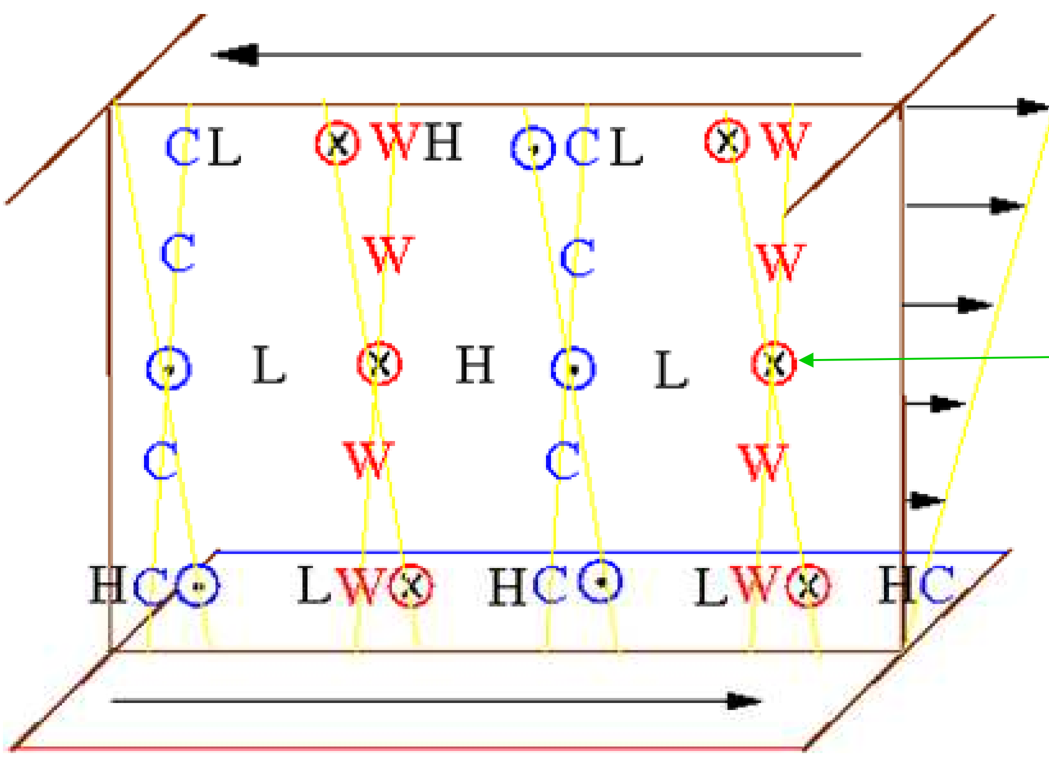
Seems to be consistent with midlatitude cyclones.

Phase relation



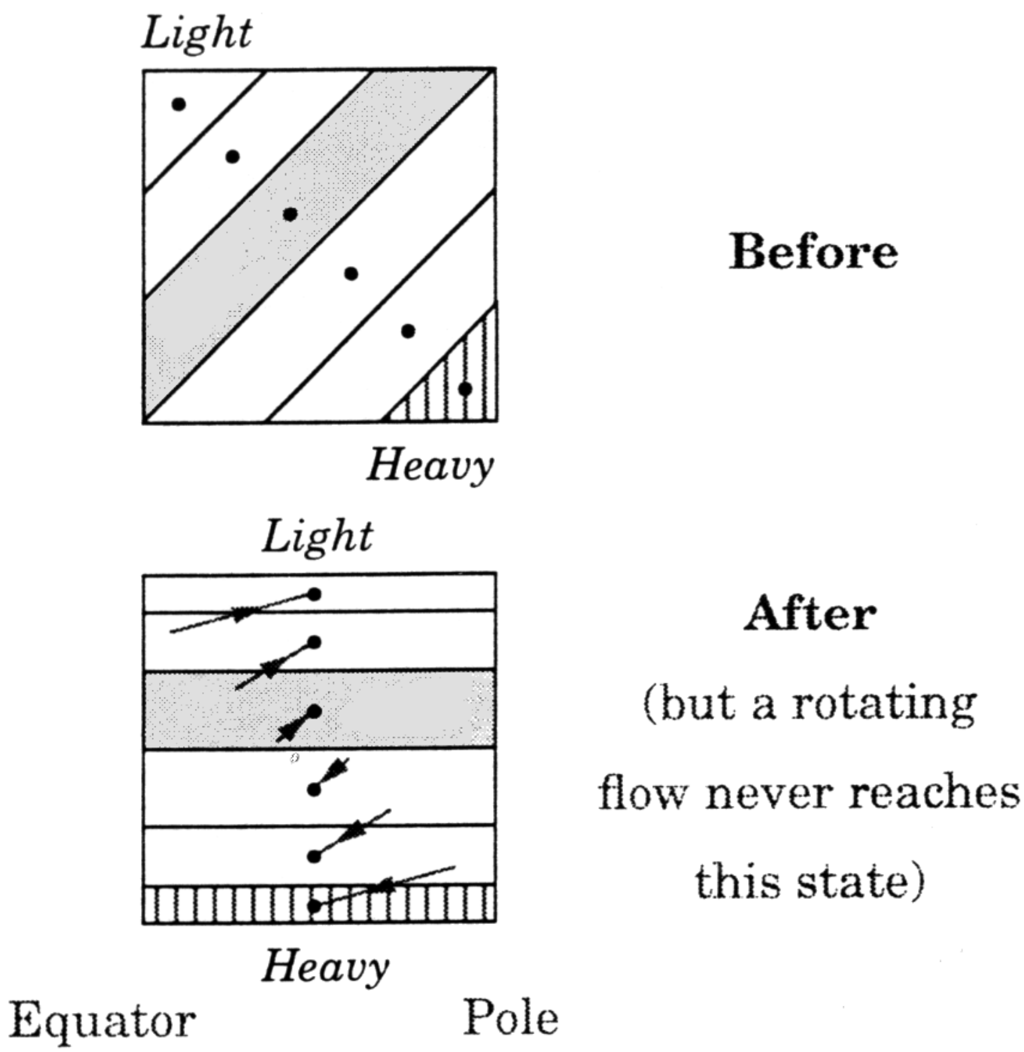
For growing mode, westward phase tilt with height

Velocity pattern increases temperature perturbation & resists tilting by shear



Warmest air moves poleward at the steering level

Density/heat transport by the growing wave



To extract energy from system, must be net poleward transfer of heat (i.e. v' must be correlated with ρ')

For wave on boundary, correlation $\overline{v'\rho'}$ exactly zero.

For growing Eady wave $\overline{v'\rho'} < 0$, $v'T' > 0$.

Laboratory experiment revisited

Figure 8.7: (Top) Baroclinic eddies in the 'eddy' regime viewed from the side. (Bottom) View from above. Eddies draw fluid from the periphery in to toward the centre at point A and vice-versa at point B. The eddies are created by the instability of the thermal wind induced by the radial temperature gradient due to the presence of the ice bucket at the centre of the tank. The diameter of the ice bucket is 15 cm.

Poleward transport of heat

Barotropic/baroclinic instabilities

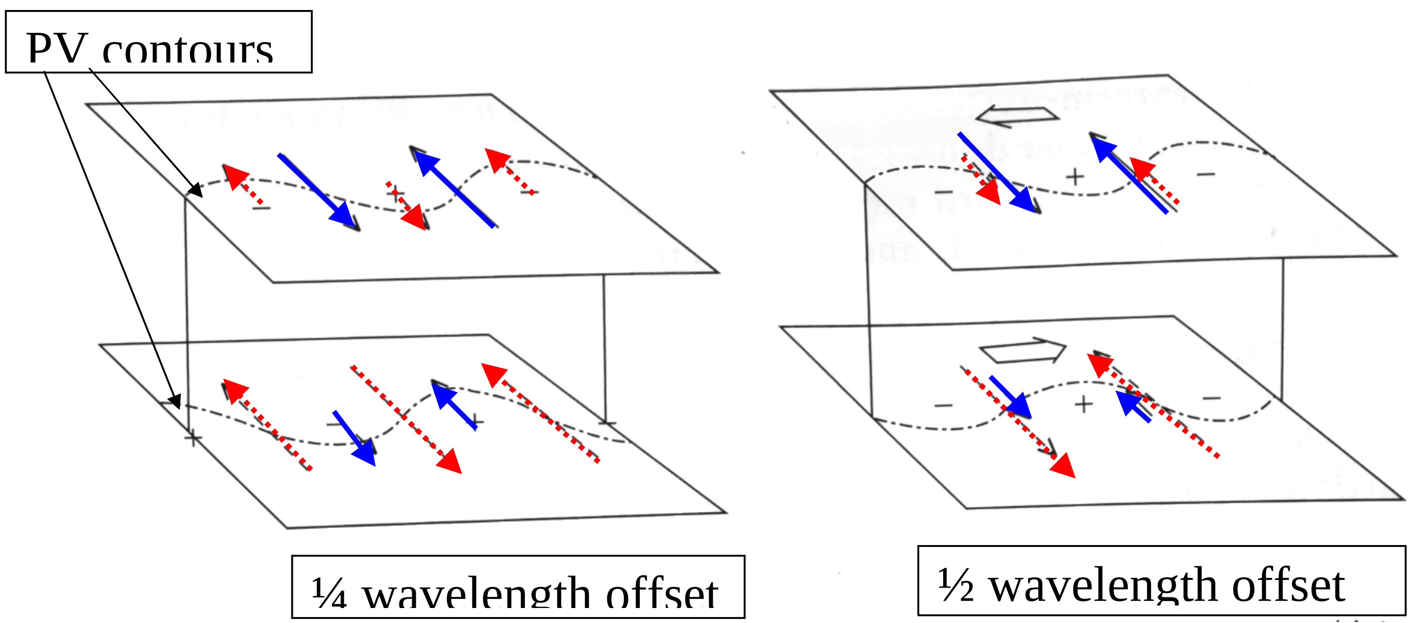
APE not sufficient for instability. What are conditions?

Baroclinic/barotropic instabilities involve Rossby wave dynamics (PV or surface temp advection)

$$\bar{q}_y = \beta - \bar{u}_{yy} - \frac{1}{\rho_0}\left(\frac{f_0^2}{N^2}\bar{u}_z\rho_0\right)_z$$

QGPV gradient in basic state

Require counter-propagating Rossby waves.



Necessary: $\bar{q}_y$, U_z upper boundary, $-U_z$ lower boundary must take both signs somewhere in flow.

Contrasting signs in vertical – baroclinic instability

in horizontal – barotropic instability

Baroclinic instability releases PE from mean flow

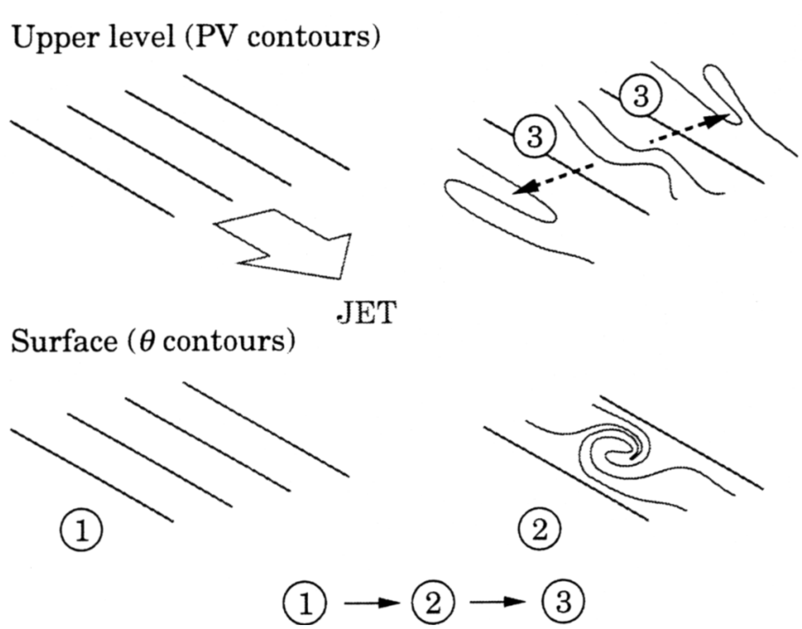
Barotropic instability releases KE from mean flow

[but note wave need not be unstable to release energy from mean flow]

What limits growth?

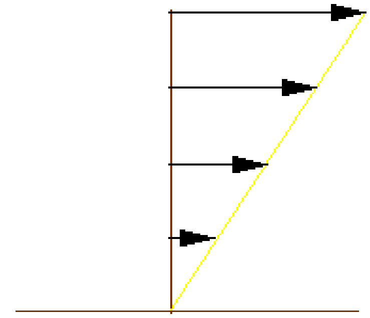
Instability of shear layer: unstable growth ceases when layer rolls up into vortices – pre-existing vorticity gradients have been considerably distrupted.

Baroclinic instability: growth ceases when temperature gradient at lower boundary is stirred up [but not necessarily the temperature gradient in the interior]



Examples

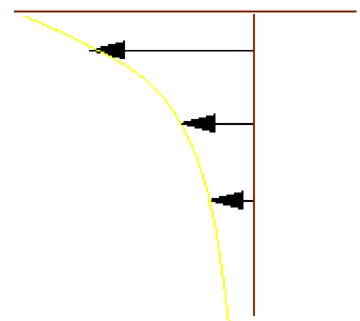
1. interior $\bar{q}_y > 0$, $\left.\dfrac{dU}{dz}\right|_{\text{bottom}} > 0$



midlatitude circulation of atmosphere

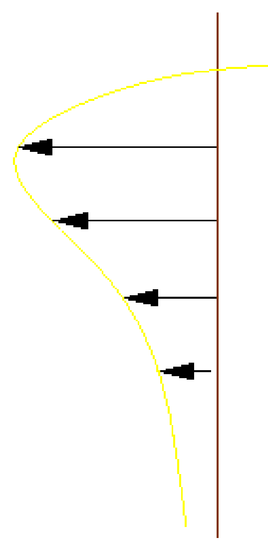
(with positive $\bar{q}_y$ concentrated at tropopause)

2. interior $\bar{q}_y > 0$, $\left.\dfrac{dU}{dz}\right|_{\text{top}} < 0$



westward flowing part of wind-driven ocean circulation

3. interior $\bar{q}_y > 0$ in some regions, < 0 in others



SH summer mesosphere? (Plumb 1983)

[possible explanation for 2-day wave?]

Summary

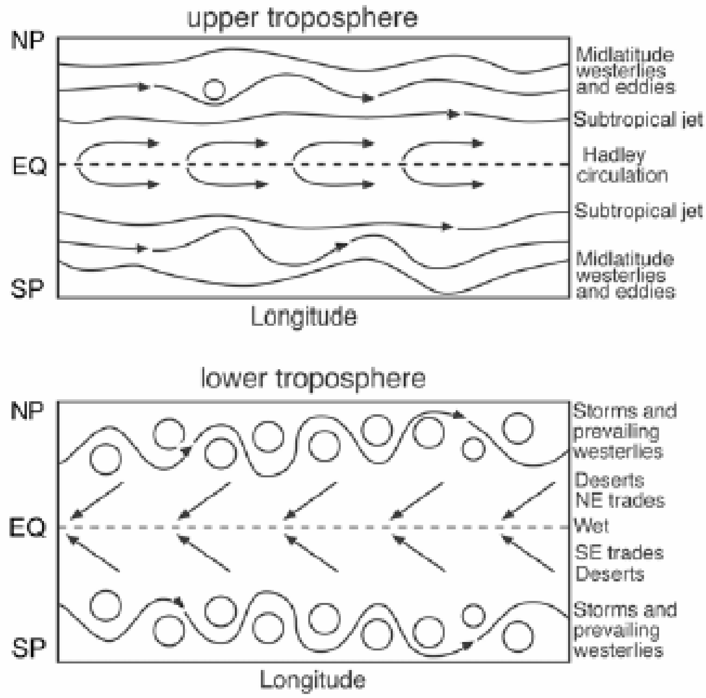


Figure 8.15: Schematic of the global distributions of atmospheric winds in the upper (top) and lower (bottom) troposphere together with the primary latitudinal climate zones.

Atmosphere must transport heat poleward to maintain temperature gradient: key role in climate.

Angular momentum must be transported poleward since westerly (*easterly*) winds at surface in midlatitudes (*tropics*).

Midlatitude baroclinic eddies transport heat & angular momentum poleward.

